Exercise 12

A fast exothermic chemical reaction was carried out in a pilot thin-film spinning disk reactor.

Pilot SDR dimensions and operational conditions:

Disk diameter	D_{pil}	0.10 m
Rotational speed	N_{pil}	1800 rpm
Liquid mass flowrate	Q_{pil}	18 kg/h

Physical properties

$\rho_L = 1000\ kg\ m^{-3}$

$\mu_L = 1 \cdot 10^{-3}\ Pa \cdot s$

$\lambda_L = 0.60\ Wm^{-1}K^{-1}$

Newtonian fluid, smooth laminar film.

Questions

1. Calculate the mean residence time in the reactor and the mean film thickness
2. Calculate the reactor size and operational conditions required to process 1000 kg/h of feed with the same product quality
3. Calculate the reactor size and operational conditions required to process 1000 kg/h of feed using five times the residence time used in the pilot unit.

Solutions

Assume smooth fully developed laminar film

1. Pilot residence time and mean film thickness:

$$\boldsymbol{\tau_{pil}} = \left(\frac{81\pi^2}{16}\frac{\nu}{\omega^2 Q_{pil}^2}\right)^{1/3} r_{pil}^{4/3} = \left(\frac{81\pi^2}{16}\frac{\nu\, r_{pil}^4}{\omega^2 Q_{pil}^2}\right)^{1/3} = \left(\frac{81\pi^2}{16}\frac{\nu}{\omega^2}\left(\frac{\boldsymbol{r_{pil}^2}}{\boldsymbol{Q_{pil}}}\right)^2\right)^{1/3} =$$

$$\mathbf{0.071\ s}$$

$$\boldsymbol{\overline{\delta}_{pil}} = \frac{\int_0^{r_{pil}}\left(\frac{3}{2\pi}\frac{\nu\, Q_{pil}}{\omega^2 r^2}\right)^{1/3} dr}{\int_0^{r_{pil}} dr} = \left(\frac{81}{2\pi}\frac{\nu}{\omega^2}\frac{\boldsymbol{Q_{pil}}}{\boldsymbol{r_{pil}^2}}\right)^{1/3} = \mathbf{90 \cdot 10^{-6}\,m}$$

2. Scale up at equal product quality:

- Keep film thickness profile and mean residence time constant → film volume increased by a factor $\frac{Q_{prod}}{Q_{pil}}$ → disk surface increased by a factor $\frac{Q_{prod}}{Q_{pil}}$ → disk diameter increased by a factor $\sqrt{\frac{Q_{prod}}{Q_{pil}}}$.
- Keep same rotational speed.
- It follows that the mean residence time, film thickness profile, mean film thickness and heat transfer coefficient will remain constant upon scaleup, since $\boldsymbol{\frac{Q}{r^2}}$ is kept constant (see equations for $\tau, \bar{\delta}, h$: all parameters constant).
- However, the shear rate, at a given vertical position $\frac{z}{\delta}$ in the film will be higher in the production unit than in the pilot unit since $\boldsymbol{Q\omega^4 r}$ increases upon scale up.

Scaleup

$$\boldsymbol{D_{prod}} = D_{pil}\sqrt{\frac{Q_{prod}}{Q_{pil}}} = \mathbf{0.745}\ \boldsymbol{m} \quad \text{and} \quad \boldsymbol{N_{prod}} = N_{pil} = \mathbf{1800}\ \boldsymbol{rpm}$$

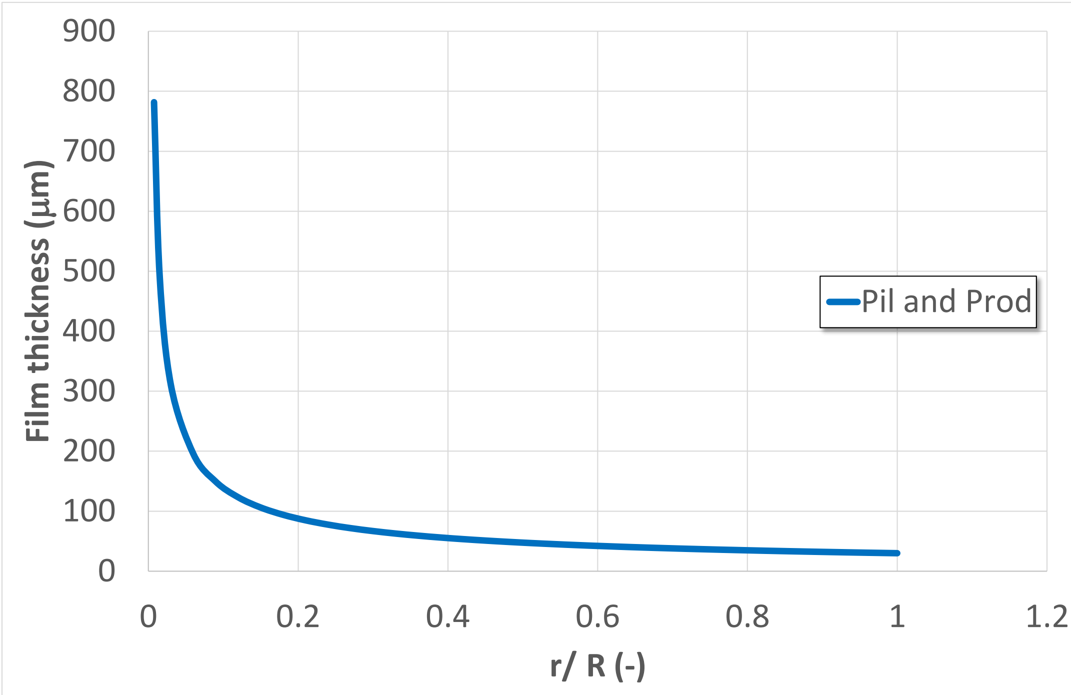
Angular velocity

$$\omega_{prod} = \omega_{pil} = 188\ rad \cdot s^{-1}$$

Film thickness

$$\bar{\delta}_{prod} = \bar{\delta}_{pil} = 90 \cdot 10^{-6} m$$

Film thickness profile:



Mean residence time

$$\tau_{prod} = \tau_{pil} = 0.071\ s$$

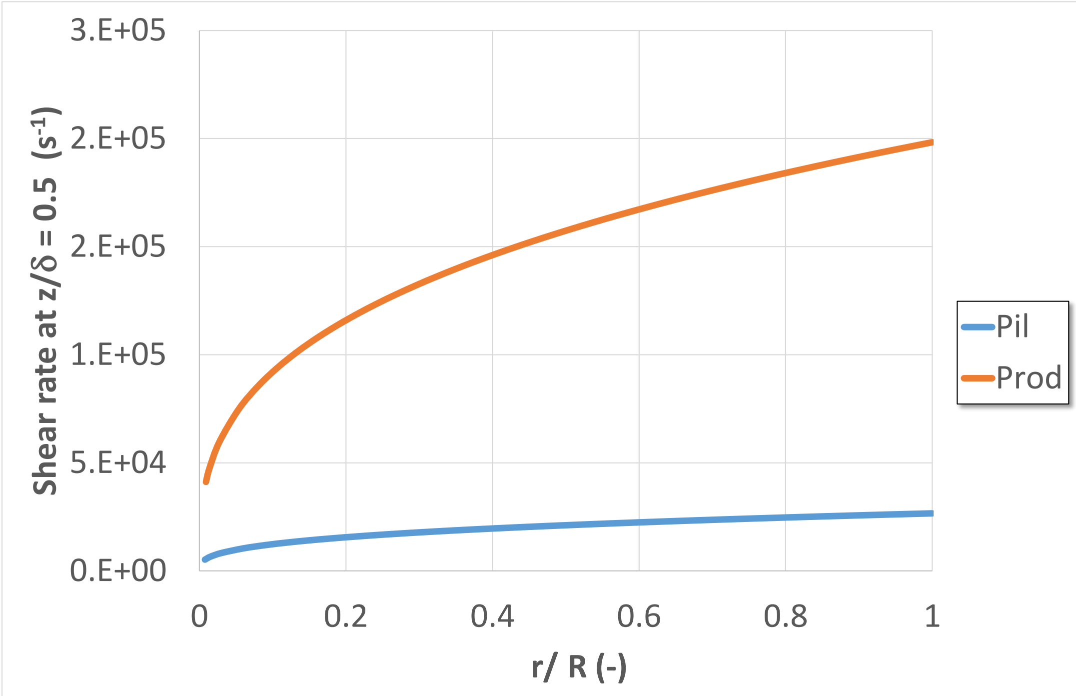
Mean heat transfer coefficient

$$h_{prod} = \frac{5}{3}\lambda\left(\frac{2\pi\ \rho\ \ \omega_{prod}^2}{3\ \mu}\frac{\boldsymbol{r_{prod}^2}}{\boldsymbol{Q_{prod}}}\right)^{\frac{1}{3}} = h_{pil} = \frac{5}{3}\lambda\left(\frac{2\pi\ \rho\ \ \omega_{pil}^2}{3\ \mu}\frac{\boldsymbol{r_{pil}^2}}{\boldsymbol{Q_{pil}}}\right)^{\frac{1}{3}}$$

$$= 33.4 \cdot 10^3\ Wm^{-2}K^{-1}$$

Shear rate profile at $\frac{z}{\delta} = 0.5$

$$\dot{\gamma}(r,z) = \frac{dv_r(r,z)}{dz} = \left(\frac{3}{2\pi}\frac{\boldsymbol{Q\omega^4 r}}{\nu^2}\right)^{\frac{1}{3}}\left(1 - \frac{z}{\delta}\right) = 0.5\left(\frac{3}{2\pi}\frac{\boldsymbol{Q\omega^4 r}}{\nu^2}\right)^{1/3}$$



3. Scale up at increased residence time:

$$\boldsymbol{\omega_{plant}} = \omega_{pil}\frac{1}{\sqrt{f}} = \frac{188}{\sqrt{5}} = \mathbf{84.3\ rad \cdot s^{-1}}$$

$$\boldsymbol{D_{plant}} = D_{pil}\sqrt{f \cdot \frac{Q_{plant}}{Q_{lab}}} = \mathbf{1.67\ m}$$

Mean residence time

$$\tau_{prod} = 5 \times \tau_{pil} = 0.35\ s$$

Mean film thickness

$$\bar{\delta}_{prod} = \bar{\delta}_{pil} = 90 \cdot 10^{-6} m$$

Mean heat transfer coefficient

$$h_{prod} = h_{pil} = 33.4 \cdot 10^3\ W m^{-2} K^{-1}$$

Shear rate profile at $\frac{z}{\delta} = 0.5$